

Original Research Paper

Optimization of Initial Alignment Process Using Low-Cost Algorithms and Sensor Reduction

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ARTICLE INFO

Article History:

Received 12 August 2025

Revised 23 February 2026

Accepted 30 April 2026

Available Online 9 June 2026

Keywords:

Strapdown inertial navigation system

Initial alignment

Sensor reduction

North finding

Cost optimization

ABSTRACT

The inertial navigation system (INS), as a stand-alone system, is capable of providing precise vehicle position, velocity, and attitude information. In recent years, strapdown inertial navigation systems (SINS) have replaced stable platform systems due to their lower cost, reduced weight, mechanical simplicity, and superior reliability. However, their performance depends heavily on accurate initialization, particularly the critical initial attitude. This paper presents a novel SINS initial alignment method that reduces the number of gyroscopes from three to one and accelerometers from three to two, significantly lowering implementation costs while maintaining adequate leveling and heading accuracy. The innovative approach combines analytical algorithms for coarse alignment and enables autonomous north-finding without expensive equipment such as high-grade gyrocompasses. Ultimately, the reduction in the number of sensors leads to cost optimization and enhanced system performance in practical applications.

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How to Cite this Article:

M. Mobtaker, H. Mohammadkarimi and M. M. Modanloo, "Optimization of initial alignment process using low-cost algorithms and sensor reduction," *Journal of Space Science and Technology*, vol. VV, no. V, pp. 1-7, 2026, <https://doi.org/10.22034/jsst.yyyy.nnnn>.



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1. INTRODUCTION

The inertial navigation system (INS) is an autonomous and self-contained navigation system capable of providing a vehicle's position, velocity, and attitude at a high sampling rate [1]. An INS utilizes inertial sensors such as accelerometers and gyroscopes to detect the vehicle's linear and angular motion [2], and it operates without relying on external information, making it resistant to external interference and jamming. Consequently, INS has been widely employed for vehicle navigation across various applications [3]. With advancements in technology, the strapdown inertial navigation system (SINS) has replaced stable platform inertial navigation systems, and extensive research has been conducted in this field [4].

In a strapdown INS, navigation states are updated through numerical integration; therefore, the initial navigation states, including position, velocity, and attitude, must be determined [5]. As a dead-reckoning system, the navigation state at each step depends on that of the previous step, making the system's performance highly dependent on the accuracy of the initial state determination [6]. Under stationary conditions, the initial position can be obtained via a Global Navigation Satellite System (GNSS), and the initial velocity can be assumed to be zero. Unlike position and velocity, obtaining the initial attitude is more challenging; thus, the objective of initial alignment is to determine the accurate transformation matrix from the body frame to the navigation frame [7].

Initial alignment depends on various factors, including the specifications of the inertial sensors, calibration performance, and environmental conditions. Additionally, the algorithms implemented during this stage significantly influence both the accuracy and speed of the alignment process. In general, depending on operational requirements, initial alignment accuracy is expected to meet a specific threshold, and alignment algorithms are evaluated based on their ability to meet the desired accuracy and speed [8].

Various methods exist for initial alignment, each suitable for specific conditions. Based on vehicle dynamics, there are stationary alignment methods and in-motion alignment methods (the latter being beyond the scope of this paper). Furthermore, alignment methods can be categorized as coarse alignment and fine alignment

depending on the level of attitude error [9].

Currently, stationary initial alignment in SINS is achieved through two consecutive stages: Analytical coarse alignment followed by fine alignment using a zero-velocity Kalman filter [10]. Coarse alignment provides a rapid initial estimate of the vehicle's attitude, typically through analytical methods based on the relationship between the Earth's rotation rate and local gravity as measured by the inertial measurement unit (IMU) [11]. This process involves three principal vectors: the gravity vector, the Earth's angular velocity vector, and their cross product [12–15]. In general, [12–14] proposed coarse alignment methods due to their shorter computation time compared to fine alignment methods, which are filter-based. However, [16] introduced a fine alignment method with faster convergence than other fine alignment approaches. Since fine alignment relies on the coarse alignment output for initial estimates, significant errors in coarse alignment results can propagate into the fine alignment process [17].

Other drawbacks of fine alignment include error growth when the degree of linearization increases, and the lack of substantial improvement over the coarse alignment accuracy. In contrast, coarse alignment has certain advantages, but it assumes ideal conditions such as a completely stationary platform and perfect sensors, disregarding errors such as misalignment.

In [18], a novel gyrocompassing method was introduced that accounts for error sources such as bias, scale factor, and misalignment. In [19], a technique for estimating dynamic position using low-cost MEMS sensors was presented. However, some studies emphasize that in high-precision applications such as ballistic missiles and satellite launch vehicles, high-grade navigation equipment like ring laser gyroscopes (RLG) or fiber-optic gyroscopes (FOG) should be employed. Nevertheless, given their high cost and the fact that such systems are often disposable, their use may be impractical in certain applications.

This study proposes a novel approach that reduces the number of gyroscopes from three to one and accelerometers from three to two, thereby significantly lowering costs while maintaining sufficient accuracy in computing leveling and heading angles. The method enables gyrocompassing without the need for expensive high-grade north finder devices.

The structure of this paper is as follows: Section 2 presents the newly proposed algorithm for determining leveling angles; Section 3 describes three methods for heading angle extraction; Section 4 discusses the simulation results; and Section 5 provides the conclusions.

2. DETERMINATION OF LEVELING ANGLES

This section presents a novel method in which two accelerometers, aligned along the x and y axes, are used to estimate the leveling (roll and pitch) angles. Under stationary ground conditions, the roll and pitch angles can be obtained following the procedure below. It is known that the acceleration measured by the accelerometers in the stationary state can be expressed as:

$$[\tilde{\mathbf{f}}]^B = [-\mathbf{g}]^B = [\mathbf{T}]^{BN} [-\mathbf{g}]^N \quad (1)$$

Equation (1) can be expanded as follows:

$$\begin{bmatrix} \tilde{f}_x \\ \tilde{f}_y \\ \tilde{f}_z \end{bmatrix} = \begin{bmatrix} \cos\psi \cos\theta & \sin\psi \cos\theta & -\sin\theta \\ \cos\psi \sin\theta \sin\varphi - \sin\psi \cos\varphi & \sin\psi \sin\theta \sin\varphi + \cos\psi \cos\varphi & \cos\theta \sin\varphi \\ \cos\psi \sin\theta \cos\varphi + \sin\psi \sin\varphi & \sin\psi \sin\theta \cos\varphi - \cos\psi \sin\varphi & \cos\theta \cos\varphi \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix} \quad (2)$$

Rewriting Eq. (2) yields:

$$\begin{bmatrix} \tilde{f}_x \\ \tilde{f}_y \\ \tilde{f}_z \end{bmatrix} = g \begin{bmatrix} \sin\theta \\ -\cos\theta \sin\varphi \\ -\cos\theta \cos\varphi \end{bmatrix} \quad (3)$$

From Eq. (3), the roll and pitch angles can be obtained as follows:

$$\varphi = \arcsin \left(\frac{\tilde{f}_y}{-g \cos \left(\arcsin \left(\frac{\tilde{f}_x}{g} \right) \right)} \right) \quad (4)$$

$$\theta = \arcsin \left(\frac{\tilde{f}_x}{g} \right) \quad (5)$$

3. DETERMINATION OF THE HEADING (YAW) ANGLE

The determination of the heading angle is more challenging than that of the leveling angles and involves more complex computations. Next, by

rotating the navigation frame (N) by an angle ψ which is unknown in this context, we obtain the intermediate frame (L). Then, through successive rotations by the angles φ and θ , derived from Eqs. (4) and (5), respectively, the transformation matrix from the intermediate frame (L) to the body frame (B) can be obtained as follows:

$$[\mathbf{T}]^{BL} = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ \sin\theta \sin\varphi & \cos\varphi & \cos\theta \sin\varphi \\ \sin\theta \cos\varphi & -\sin\varphi & \cos\theta \cos\varphi \end{bmatrix} \quad (6)$$

By transforming the angular velocity vectors measured by the gyroscopes from the body frame to the intermediate frame (L), we obtain the following expression:

$$[\tilde{\boldsymbol{\omega}}^{BL}]^L = [\tilde{\mathbf{T}}]^{BL} [\tilde{\boldsymbol{\omega}}^{BL}]^B \rightarrow \begin{bmatrix} \omega^{EL} \cos\lambda \cos\psi \\ -\omega^{EL} \cos\lambda \sin\psi \\ -\omega^{EL} \sin\lambda \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \sin\varphi & \sin\theta \cos\varphi \\ 0 & \cos\varphi & -\sin\varphi \\ -\sin\theta & \cos\theta \sin\varphi & \cos\theta \cos\varphi \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \quad (7)$$

Expanding Equation (7), we obtain:

$$\omega^{EL} \cos\lambda \cos\psi = \omega_x \cos\theta + \omega_y \sin\theta \sin\varphi + \omega_z \sin\theta \cos\varphi \quad (8)$$

$$-\omega^{EL} \cos\lambda \sin\psi = \omega_y \cos\varphi - \omega_z \sin\varphi \quad (9)$$

$$-\omega^{EL} \sin\lambda = -\omega_x \sin\theta + \omega_y \cos\theta \sin\varphi + \omega_z \cos\theta \cos\varphi \quad (10)$$

In Eqs (8), (9), and (10), we observe that there are four unknowns: ω_x , ω_y , ω_z and ψ . To solve these equations, one of the unknowns must be considered known. Naturally, the heading angle ψ cannot be assumed as known since it is precisely the parameter we want to calculate. Therefore, the number of unknowns must be reduced by measuring one of the angular velocities. For this purpose, a rate gyro aligned along one of the axes can be used. In the following, the use of gyroscopes in each of the x, y and z channels will be examined separately.

3.1. Gyroscope Deployment along the x-Axis

In this case, using Eqs (8), (9), and (10), we encounter three algebraic equations with three unknowns ω_y , ω_z , and ψ . This system can be rewritten as follows:

$$\omega_y \sin \theta \sin \varphi + \omega_z \sin \theta \cos \varphi - \omega^{\text{EI}} \cos \lambda \cos \psi = -\tilde{\omega}_x \cos \theta \quad (11)$$

$$\omega_y \cos \varphi - \omega_z \sin \varphi + \omega^{\text{EI}} \cos \lambda \sin \psi = 0 \quad (12)$$

$$\omega_y \cos \theta \sin \varphi + \omega_z \cos \theta \cos \varphi = -\omega^{\text{EI}} \sin \lambda + \tilde{\omega}_x \sin \theta \quad (13)$$

Assuming $\varphi \neq 0$, we have from Eq. 12:

$$\omega_z = \frac{\omega_y \cos \varphi + \omega^{\text{EI}} \cos \lambda \sin \psi}{\sin \varphi} \quad (14)$$

Now, by substituting Eq. 14 into Eq. 13, we obtain:

$$\omega_y \cos \theta + \omega^{\text{EI}} \cos \lambda \cos \theta \cos \varphi \sin \psi = \tilde{\omega}_x \sin \theta \sin \varphi - \omega^{\text{EI}} \sin \lambda \sin \varphi \quad (15)$$

Assuming $\theta \neq 0$, we have:

$$\omega_y = \frac{\tilde{\omega}_x \sin \theta \sin \varphi - \omega^{\text{EI}} \sin \lambda \sin \varphi - \omega^{\text{EI}} \cos \lambda \cos \theta \cos \varphi \sin \psi}{\cos \theta} \quad (16)$$

By substituting Eq. 16 into Eq. 11, we have:

$$0.25 \sin 2\theta \sin 2\varphi \sin \psi + \cos \theta \cos \psi - \frac{\sin \theta \cos \theta \cos \varphi}{\omega^{\text{EI}} \cos \lambda} \omega_z = \frac{\tilde{\omega}_x (1 - \sin^2 \theta \cos^2 \varphi)}{\omega^{\text{EI}} \cos \lambda} + \tan \lambda \sin^2 \varphi \sin \theta \quad (17)$$

This Eq. does not have an explicit analytical solution; therefore, the use of a gyroscope on the x-axis is not recommended.

3.2. Gyroscope Deployment along the y-Axis

In this case, Eqs. (8), (9), and (10) contain the unknowns ω_x , ω_z and ψ . By rewriting Eqs. (8), (9), and (10), we obtain:

$$\omega_x \cos \theta + \omega_z \sin \theta \cos \varphi - \omega^{\text{EI}} \cos \lambda \cos \psi = -\tilde{\omega}_y \sin \theta \sin \varphi \quad (18)$$

$$-\omega_z \sin \varphi + \omega^{\text{EI}} \cos \lambda \sin \psi = -\tilde{\omega}_y \cos \varphi \quad (19)$$

$$-\omega_x \sin \theta + \omega_z \cos \theta \cos \varphi = -\omega^{\text{EI}} \sin \lambda - \tilde{\omega}_y \cos \theta \sin \varphi \quad (20)$$

Assuming $\varphi \neq 0$, we have from Eq. (18):

$$\omega_z = \frac{\omega^{\text{EI}} \cos \lambda \sin \psi + \tilde{\omega}_y \cos \varphi}{\sin \varphi} \quad (21)$$

By substituting Eq. (21) into Eq. (20), we have:

$$-\omega_x \sin \theta \sin \varphi + \omega^{\text{EI}} \cos \lambda \cos \theta \cos \varphi \sin \psi = -\tilde{\omega}_y \cos \theta - \omega^{\text{EI}} \sin \lambda \sin \varphi \quad (22)$$

Equation (22) can also be written as follows, assuming $\theta \neq 0$:

$$\omega_x = \frac{\omega^{\text{EI}} \cos \lambda \cos \theta \cos \varphi \sin \psi + \tilde{\omega}_y \cos \theta + \omega^{\text{EI}} \sin \lambda \sin \varphi}{\sin \theta \sin \varphi} \quad (23)$$

By substituting Eq. (23) into Eq. (18), we obtain:

$$\cos^2 \theta \cos \varphi \sin \psi - \sin \theta \sin \varphi \cos \psi + \frac{\sin^2 \theta \cos \varphi \sin \varphi}{\omega^{\text{EI}} \cos \lambda} \omega_z = \frac{-\tilde{\omega}_y (1 - \sin^2 \varphi)}{\omega^{\text{EI}} \cos \lambda} - \tan \lambda \sin \varphi \cos \theta \quad (24)$$

Similar to the previous case, the above equation does not have an explicit analytical solution; therefore, the use of a gyroscope on the y-axis is also not suitable and is not recommended.

3.3. Gyroscope Deployment along the z-Axis

desirable result is achieved by measuring ω_z . In this case, the unknowns are ω_x , ω_y , and ψ . Accordingly, Eqs. (8), (9), and (10) can be expressed as follows:

$$\omega_x \cos \theta + \omega_y \sin \theta \sin \varphi - \omega^{\text{EI}} \cos \lambda \cos \psi = -\tilde{\omega}_z \sin \theta \cos \varphi \quad (25)$$

$$\omega_y \cos \varphi + \omega^{\text{EI}} \cos \lambda \sin \psi = +\tilde{\omega}_z \sin \varphi \quad (26)$$

$$-\omega_x \sin \theta + \omega_y \cos \theta \sin \varphi = -\omega^{\text{EI}} \sin \lambda - \tilde{\omega}_z \cos \theta \cos \varphi \quad (27)$$

From Eq. (27), assuming $\theta \neq 0$, it can be written as:

$$\omega_x = \frac{\omega_y \cos \theta \sin \varphi + \omega^{\text{EI}} \sin \lambda + \tilde{\omega}_z \cos \theta \cos \varphi}{\sin \theta} \quad (28)$$

By substituting Eq. (28) into Eq. (25), we have:

$$\omega_y \sin \varphi - \omega^{\text{EI}} \cos \lambda \sin \theta \cos \psi = -\omega^{\text{EI}} \sin \lambda \cos \theta - \tilde{\omega}_z \cos \varphi \quad (29)$$

Now, with some simplification and assuming $\varphi \neq 0$, Eq. (30) is obtained:

$$\omega_y = \frac{-\omega^{\text{EI}} \sin \lambda \cos \theta - \tilde{\omega}_z \cos \varphi + \omega^{\text{EI}} \cos \lambda \sin \theta \cos \psi}{\sin \varphi} \quad (30)$$

By substituting Eq. (30) into Eq. (26), we obtain:

$$\omega^{\text{EI}} \cos \lambda \sin \theta \cos \varphi \cos \psi + \omega^{\text{EI}} \cos \lambda \sin \varphi \sin \psi = \tilde{\omega}_z + \omega^{\text{EI}} \sin \lambda \cos \varphi \cos \theta \quad (31)$$

Rewriting Eq. (31) yields:

$$\sin \varphi \sin \psi + \sin \theta \cos \varphi \cos \psi = \frac{\tilde{\omega}_z}{\omega^{\text{EI}} \cos \lambda} + \tan \lambda \cos \varphi \cos \theta \quad (32)$$

In Eq. (32), the condition for the existence of a solution is as follows:

$$\sin^2 \varphi + \sin^2 \theta \cos^2 \varphi \geq \left(\frac{\tilde{\omega}_z}{\omega^{\text{EI}} \cos \lambda} + \tan \lambda \cos \varphi \cos \theta \right)^2 \quad (33)$$

The value of ω_z is:

$$\tilde{\omega}_z = (\cos \psi \sin \theta \cos \varphi + \sin \psi \sin \varphi) \cos \lambda \omega^{\text{EI}} - \cos \theta \cos \varphi \sin \lambda \omega^{\text{EI}} \quad (34)$$

Hence, we obtain:

$$\begin{aligned} \frac{\tilde{\omega}_z}{\omega^{\text{EI}} \cos \lambda} + \tan \lambda \cos \varphi \cos \theta = \\ (\cos \psi \sin \theta \cos \varphi + \sin \psi \sin \varphi) + \tan \lambda \cos \theta \cos \varphi \\ - \tan \lambda \cos \theta \cos \varphi = (\cos \psi \sin \theta \cos \varphi + \sin \psi \sin \varphi) \end{aligned} \quad (35)$$

By substituting Eq. (35) into Eq. (33), the condition for the existence of a solution is simplified as:

$$\sin^2 \varphi + \sin^2 \theta \cos^2 \varphi \geq \left(\frac{\sin \psi^2 \sin \varphi^2 + \cos \psi^2 \sin \theta^2 \cos \varphi^2}{2 \sin \psi \sin \varphi \cos \psi \sin \theta \cos \varphi} \right) \quad (36)$$

Equation (36) can also be rewritten as follows:

$$\sin^2 \varphi \cos^2 \psi + \sin^2 \theta \cos^2 \varphi \sin^2 \psi \geq 2 \sin \varphi \cos \psi \sin \theta \cos \varphi \sin \psi \quad (37)$$

The obtained equation can be expressed as the following identity:

$$(\sin \varphi \cos \psi - \sin \theta \cos \varphi \sin \psi)^2 \geq 0 \quad (38)$$

Therefore, Eq. (32) always has a solution. To solve Eq. (32), the following approach is taken:

$$\begin{cases} a = \sin \varphi \\ b = \cos \varphi \sin \theta \\ c = \frac{\tilde{\omega}_z}{\omega^{\text{EI}} \cos \lambda} + \tan \lambda \cos \varphi \cos \theta \\ \tan \zeta = \frac{b}{a} \end{cases} \quad (39)$$

Subsequently, we obtain:

$$\begin{aligned} \sin(\psi + \zeta) &= \frac{c}{a} \cos \zeta \rightarrow \psi = \sin^{-1} \left(\frac{c}{a} \cos \zeta \right) - \zeta \rightarrow \\ \psi &= \sin^{-1} \left(\mp \frac{c}{\sqrt{a^2 + b^2}} \right) - \tan^{-1} \left(\frac{b}{a} \right) \end{aligned} \quad (40)$$

It can be observed that the heading angle is calculated according to Eq. (40). Since an analytical solution for extracting the heading angle has been obtained in this section, the use of a gyroscope on the z-axis is deemed appropriate. This analytical solution prevents errors associated with numerical methods and results in higher final accuracy in estimating the heading angle.

4. SIMULATION

After introducing the new method based on three sensors—accelerometers on the x and y axes and a gyroscope on the z-axis—it is essential to simulate this approach and compare it with conventional existing methods to validate the equations and verify their accuracy.

The conventional coarse alignment method, as described in reference [2], involves the following calculations:

$$[\mathbf{T}]^{\text{BN}} \approx \begin{bmatrix} \frac{\tilde{\omega}_x(\tilde{f}_y^2 + \tilde{f}_z^2) - \tilde{f}_x(\tilde{\omega}_y \tilde{f}_y + \tilde{\omega}_z \tilde{f}_z)}{g^2 \omega^{\text{EI}} \cos \lambda} & \frac{\tilde{f}_z \tilde{\omega}_y - \tilde{f}_y \tilde{\omega}_z}{g \omega^{\text{EI}} \cos \lambda} & \frac{-\tilde{f}_x}{g} \\ \frac{\tilde{\omega}_y(\tilde{f}_x^2 + \tilde{f}_z^2) - \tilde{f}_y(\tilde{\omega}_x \tilde{f}_x + \tilde{\omega}_z \tilde{f}_z)}{g^2 \omega^{\text{EI}} \cos \lambda} & \frac{\tilde{f}_x \tilde{\omega}_z - \tilde{f}_z \tilde{\omega}_x}{g \omega^{\text{EI}} \cos \lambda} & \frac{-\tilde{f}_y}{g} \\ \frac{\tilde{\omega}_z(\tilde{f}_x^2 + \tilde{f}_y^2) - \tilde{f}_z(\tilde{\omega}_x \tilde{f}_x + \tilde{\omega}_y \tilde{f}_y)}{g^2 \omega^{\text{EI}} \cos \lambda} & \frac{\tilde{f}_y \tilde{\omega}_x - \tilde{f}_x \tilde{\omega}_y}{g \omega^{\text{EI}} \cos \lambda} & \frac{-\tilde{f}_z}{g} \end{bmatrix} \quad (41)$$

As previously mentioned and evident from Eq. (41), conventional methods utilize all three accelerometers and three gyroscopes for the initial alignment process. Currently, the attitude and heading angles can be extracted using the following equations:

$$\begin{cases} \psi = \arctan(T_{12}/T_{11}) \\ \theta = \arcsin(-T_{13}) \\ \varphi = \arctan(T_{23}/T_{33}) \end{cases} \quad (42)$$

In this simulation, by applying bias, scale factor, and misalignment errors, both initial alignment algorithms were executed over 50 different random scenarios for attitude and position, as well as 50 datasets corresponding to accelerometer and gyroscope errors for each scenario. Several examples of these are presented in Tables 1 and 2.

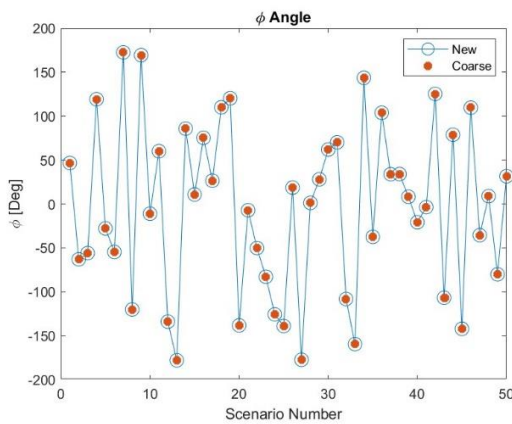
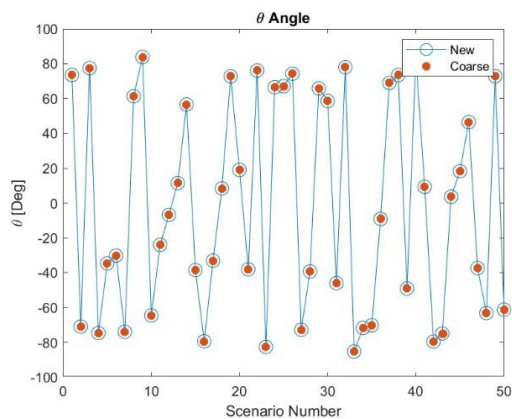
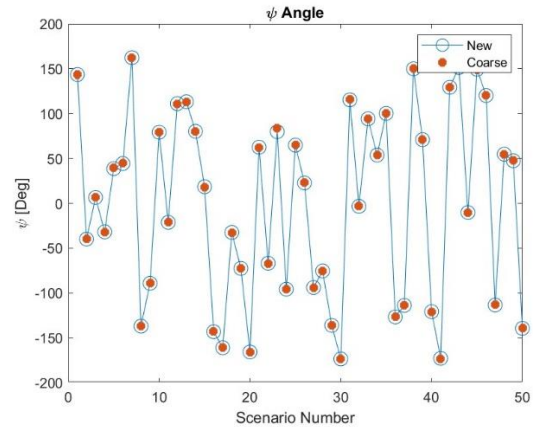
Table 1. Some of the attitude and position data in different scenarios

No.	ϕ (deg)	θ (deg)	ψ (deg)	λ (deg)
1	46.613	73.551	143.531	52.063
2	-62.945	-71.000	-39.908	52.703
3	-55.853	77.477	6.554	7.125
4	119.151	-74.700	-32.002	-77.400
5	-27.888	-34.659	39.528	40.374

Table 2. Some of the bias and drift data in different scenarios

No.	$b_x (\mu g)$	$b_y (\mu g)$	$b_z (\mu g)$	$d_x (^\circ / hr)$	$d_y (^\circ / hr)$	$d_z (^\circ / hr)$
1	55	123	147	0.012	0.009	0.005
2	133	148	85	0.014	0.008	0.005
3	119	83	130	0.005	0.006	0.008
4	72	106	100	0.015	0.010	0.006
5	73	99	140	0.014	0.015	0.008

Using the aforementioned data, simulations of the derived equations were conducted across 50 scenarios. The results are presented with the new algorithm denoted as "New" and the conventional coarse alignment algorithm labeled as "Coarse".

**Fig. 1.** Simulation of roll angle estimation.**Fig. 2.** Simulation of pitch angle estimation.**Fig. 3.** Simulation of yaw angle estimation.

It can be observed that the developed low-cost algorithm, which utilizes fewer sensors (two accelerometers and one gyroscope), achieves accuracy comparable to conventional initial alignment methods that employ three accelerometers and three gyroscopes. Therefore, the conducted simulations validate the correctness and reliability of the derived equations in the new algorithm.

5. CONCLUSION

This study addresses the importance and high cost of initial alignment in certain applications and proposes a novel low-cost method for this process. The new algorithm was evaluated through simulations and compared with conventional coarse alignment methods. To closely reflect real-world conditions, simplifications were minimized, and errors such as bias, misalignment, and scale factor were incorporated into the simulations. A distinctive feature of the proposed algorithm is the reduction in the number of gyroscopes and accelerometers, which significantly lowers costs. Specifically, only one gyroscope along the z-axis and one accelerometer on each of the x and y axes are employed. The simulation results demonstrate that the accuracy of this algorithm in attitude determination is excellent and acceptable, making it a viable alternative to existing methods. Additionally, this algorithm can be used for the gyrocompassing process. By calculating the roll and pitch angles through the proposed algorithm or an inclinometer with high accuracy, and utilizing only one relatively expensive gyroscope on the z-axis, the heading angle can be accurately computed, enabling precise gyrocompassing.

CONFLICT OF INTEREST

The authors declare that they have no conflict of interest.

REFERENCES

- [1] X. Xu, J. Gui, Y. Sun, Y. Yao, and T. Zhang, "A robust in-motion alignment method with inertial sensors and Doppler velocity log," *IEEE Transactions on Instrumentation and Measurement*, vol. 70, 2020, Art. no. 8500413, <https://doi.org/10.1109/TIM.2020.3011873>.
- [2] E. Bekir, *Introduction to Modern Navigation Systems*, World Scientific, 2007.
- [3] Y. Huang, Z. Zhang, S. Du, Y. Li, and Y. Zhang, "A high-accuracy GPS-aided coarse alignment method for MEMS-based SINS," *IEEE Transactions on Instrumentation and Measurement*, vol. 69, no. 10, pp. 7914–7932, 2020, <https://doi.org/10.1109/TIM.2020.2983578>.
- [4] T. Zou, L. Wang, T. Zhu, X. Zhai, and C. Liu, "Fine alignment algorithm of regular tetrahedral redundant strapdown inertial navigation system base on Kalman filter," in *2022 2nd International Conference on Electrical Engineering and Mechatronics Technology (ICEEMT)*, Hangzhou, China, 2022, <https://doi.org/10.1109/ICEEMT56362.2022.9862624>.
- [5] Y. G. Zhang, Y. L. Huang, Z. M. Wu, and N. Li, "Moving state marine SINS initial alignment based on high degree CKF," *Mathematical Problems in Engineering*, vol. 2014, no. 1, 2014, Art. no. 546107, <https://doi.org/10.1155/2014/546107>.
- [6] Y. Zhang, Y. Huang, N. Li, and Z. Wu, "SINS initial alignment based on fifth-degree cubature Kalman filter," in *IEEE International Conference on Mechatronics and Automation*, Takamatsu, Japan, 2013, <https://doi.org/10.1109/ICMA.2013.6617952>.
- [7] H. Y. Liu, S. Tsai, M. L. Tsai, and K. W. Chiang, "An initial alignment method of inertial navigation system for the static state," *The International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences*, vol. 43, pp. 227–233, 2022, <https://doi.org/10.5194/isprs-archives-XLIII-B1-2022-227-2022>.
- [8] H. Jameian and B. Safarinejadian, "Overcoming the settling time challenge of initial alignment of strapdown inertial navigation system by a novel solution," *IET Radar, Sonar & Navigation*, vol. 15, no. 4, pp. 419–430, 2021, <https://doi.org/10.1049/rsn2.12045>.
- [9] M. Reinstein, "Evaluation of fine alignment algorithm for inertial navigation," *Przegląd Elektrotechniczny*, vol. 87, pp. 255–258, 2011.
- [10] Z. Fan, H. Chai, X. Liang, and H. Wang, "Experimental research on shipborne SINS rapid mooring alignment with variance-constraint Kalman filter and GNSS position updates," *Sensors*, vol. 24, no. 11, 2024, Art. no. 3487, <https://doi.org/10.3390/s24113487>.
- [11] A. Frutuoso, F. O. Silva, and E. A. de Barros, "Performance evaluation of coarse alignment methods for autonomous underwater vehicles in mooring conditions," *Ocean Engineering*, vol. 282, 2023, Art. no. 114991, <https://doi.org/10.1016/j.oceaneng.2023.114991>.
- [12] F. O. Silva, W. C. Leite Filho, and E. M. Hemerly, "Design of a stationary self-alignment algorithm for strapdown inertial navigation systems," in *IFAC-PapersOnLine*, vol. 48, no. 9, pp. 55–60, 2015, <https://doi.org/10.1016/j.ifacol.2015.08.059>.
- [13] F. O. Silva, E. M. Hemerly, and W. C. L. Filho, "Error analysis of analytical coarse alignment formulations for stationary SINS," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 52, no. 4, pp. 1777–1796, 2016, <https://doi.org/10.1109/TAES.2016.7738355>.
- [14] F. O. Silva, E. M. Hemerly, W. C. L. Filho, and H. K. Kuga, "A fast in-field coarse alignment and bias estimation method for stationary intermediate-grade IMUs," *IEEE Transactions on Instrumentation and Measurement*, vol. 67, no. 4, pp. 831–838, 2018, <https://doi.org/10.1109/TIM.2017.2789138>.
- [15] M. Wu, Y. Wu, X. Hu, and D. Hu, "Optimization-based alignment for inertial navigation systems: Theory and algorithm," *Aerospace Science and Technology*, vol. 15, no. 1, pp. 1–17, Jan. 2011, <https://doi.org/10.1016/j.ast.2010.05.004>.
- [16] J. Li, Y. Li, and B. Liuxs, "Fast fine initial self-alignment of INS in erecting process on stationary base," *Journal of Navigation*, vol. 71, no. 3, pp. 697–710, 2018, <https://doi.org/10.1017/S037346331700090X>.
- [17] X. Xu, D. Xu, T. Zhang, and H. Zhao, "In-motion coarse alignment method for SINS/GPS using position loci," *IEEE Sensors Journal*, vol. 19, no. 10, pp. 3930–3938, 2019, <https://doi.org/10.1109/JSEN.2019.2896274>.
- [18] M. Hemmati and M. A. Massoumnia, "Processing algorithm for a strapdown gyrocompass," *Scientia Iranica*, vol. 19, no. 3, pp. 774–781, 2012, <https://doi.org/10.1016/j.scient.2012.01.002>.
- [19] R. H. Rogne, T. H. Bryne, T. I. Fossen, and T. A. Johansen, "On the usage of low-cost MEMS sensors, strapdown inertial navigation, and nonlinear estimation techniques in dynamic positioning," *IEEE Journal of Oceanic Engineering*, vol. 46, no. 1, pp. 24–39, 2021, <https://doi.org/10.1109/JOE.2020.2967094>.