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Original Research Paper

Online Algebraic Compensation of Inertial Sensor Biases in Reference-Dependent Navigation Systems

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ABSTRACT

Accurate calibration of inertial sensor biases is critical for preventing accumulated error growth in inertial navigation systems (INS). Conventional approaches, such as Kalman filtering and its variants, often involve significant computational complexity, dependence on precise noise modeling, and require extensive parameter tuning to ensure stability and convergence. To address these challenges, this paper presents a novel online algebraic calibration method that reformulates the INS error dynamics to directly isolate and express accelerometer and gyroscope bias terms in a closed-form manner. The proposed algorithm estimates biases by solving a deterministic system of linear equations constructed from the error states between a reference and a dependent navigation system, thereby completely eliminating the need for iterative filtering or probabilistic updates. Comprehensive simulation studies under both stationary and cruise flight conditions demonstrate the method's high accuracy, rapid convergence, and robustness against measurement noise. The results further highlight its excellent numerical stability and adaptability to time-varying sensor characteristics. Overall, the proposed approach achieves precise and consistent bias estimation with minimal computational cost, providing a fast, stable, and resource-efficient alternative for real-time implementation in safety-critical applications such as aviation, autonomous vehicles, and robotics.

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NOMENCLATURE

$\varepsilon\lambda$	Latitude error
εh	Altitude error
εv_{NED}	Velocity error in NED coordinates
$\varepsilon\phi$	Roll angle error
$\varepsilon\theta$	Pitch angle error
$\varepsilon\psi$	Yaw angle error
δf_{xyz}	Accelerometer bias in body coordinates
$\delta\omega_{xyz}$	Gyroscope bias in body coordinates

1. INTRODUCTION

Inertial Navigation Systems (INS) play a central role in modern navigation by integrating measurements from accelerometers and gyroscopes to autonomously estimate position, velocity, and orientation. However, the performance of an INS is inherently influenced by sensor errors—particularly biases—that accumulate over time and lead to a gradual degradation of accuracy. Their adverse effects are amplified during successive integrations, resulting in growing position estimation errors [1,2].

Therefore, compensating for such errors is essential to maintaining the reliability and accuracy of INS, especially in safety-critical applications such as autonomous vehicles, aerospace systems, and robotics, where state estimation errors can have catastrophic consequences [3].

Conventional error compensation approaches are typically based on calibration procedures performed under controlled conditions, which are unsuitable for dynamic operational environments that demand real-time compensation. To address this, methods like the Kalman Filter (KF) and its variants (e.g., EKF[1], UKF[2]) have become standard [4,5]. Extensive developments in this domain include quaternion-based EKF formulations [6], unscented filtering schemes for attitude estimation [7], and particle filtering methods for non-Gaussian scenarios [8], alongside detailed observability analyses for inertial sensor configurations [9]. However, a significant shortcoming of these filter-based approaches is their inherent dependency on recursive iterations and the careful tuning of noise covariance matrices, which can lead to convergence issues and requires considerable expert knowledge for optimal performance. Similarly, while autoregressive (AR) models can describe error temporal accumulation [10], they often lack the robustness for generalizing across diverse dynamic conditions.

More recently, data-driven strategies, particularly deep learning, have emerged to model complex sensor error dynamics directly from data [11]. Optimized frameworks like the LS-SVM[3] with CPSO[4] [12] have demonstrated promising accuracy. Nevertheless, a primary limitation of these methods is their substantial computational demand and their reliance on large, representative datasets for training, making them less practical resource-limited applications where training data is scarce. Moreover, while geometric methods such as nonlinear complementary filters [13] offer deterministic alternatives on Lie groups, they still rely on continuous feedback tuning.

This paper proposes a novel algebraic method to address these specific shortcomings.

The primary novelty of this work lies in its fundamental departure from iterative and probabilistic estimation paradigms, drawing inspiration from algebraic identification methods [14]. Unlike Kalman filter-based approaches, which require recursive updates and careful tuning of process and measurement noise covariance matrices, our method provides a direct, deterministic solution. Furthermore, in contrast to emerging data-driven techniques that demand extensive training data, the proposed algebraic approach operates based on instantaneous error dynamics, requiring no prior training or complex parameter tuning. This core innovation—the algebraic isolation of bias terms from the navigation error equations—transforms a typically recursive estimation problem into a solvable system of linear equations, resulting in a computationally lightweight and highly efficient algorithm capable of real-time bias estimation without the convergence delays inherent in iterative filters.

The remainder of this paper is organized as follows. Section 2 presents the inertial navigation error equations and their relationship to accelerometer and gyroscope biases. Section 3 describes the modeling approach in Simulink. Section 4 validates the derived equations through simulations in two common scenarios. Finally, Sections 5 and 6 present the conclusions and references, respectively.

1. Extended Kalman Filter (EKF)
2. Unscented Kalman Filter (UKF)
3. Least Squares Support Vector Machine
4. Chaotic Particle Swarm Optimization

2. INERTIAL NAVIGATION ERROR EQUATIONS

This section details the mathematical derivation of the proposed algebraic method for bias compensation. The core of the approach is to transform the standard inertial navigation error equations into a solvable algebraic system.

The process begins with the fundamental error dynamics for velocity and attitude. By systematically rearranging these equations, the accelerometer and gyroscope bias terms are algebraically isolated. This formulation allows for the direct calculation of the biases, providing a computationally efficient alternative to iterative or filter-based estimation techniques.

2.1 Derivation of Algebraic Bias Equations

Based on reference [15], the velocity error equations are expressed as Eq. 1.

$$
\begin{aligned}
\varepsilon\dot{v}_n &= T_{11}^{NB}\delta f_x + T_{12}^{NB}\delta f_y + T_{13}^{NB}\delta f_z \\
&\quad + f_e\varepsilon\psi - f_d\varepsilon\theta - \left(\frac{v_e{}^2\sec^2\lambda}{R_e} + 2\omega_n v_e\right)\varepsilon\lambda \\
&\quad + \frac{\left(v_e{}^2\tan\lambda - v_n v_d\right)}{R_e{}^2}\varepsilon h + \frac{v_d}{R_e}\varepsilon v_n \\
&\quad + \left(2\omega_d - \frac{2v_e\tan\lambda}{R_e}\right)\varepsilon v_e + \frac{v_n}{R_e}\varepsilon v_d \\
\varepsilon\dot{v}_e &= T_{21}^{NB}\delta f_x + T_{22}^{NB}\delta f_y + T_{23}^{NB}\delta f_z \\
&\quad - f_n\varepsilon\psi + f_d\varepsilon\phi + \left(\frac{v_e v_n\sec^2\lambda}{R_e} + 2\omega_n v_n + 2\omega_d v_d\right)\varepsilon\lambda \\
&\quad - \frac{\left(v_e v_n\tan\lambda + v_e v_d\right)}{R_e{}^2}\varepsilon h + \left(\frac{v_e\tan\lambda}{R_e} - 2\omega_d\right)\varepsilon v_n \\
&\quad + \frac{\left(v_n\tan\lambda + v_d\right)}{R_e}\varepsilon v_e + \left(\frac{v_e}{R_e} + 2\omega_n\right)\varepsilon v_d \\
\varepsilon\dot{v}_d &= T_{31}^{NB}\delta f_x + T_{32}^{NB}\delta f_y + T_{33}^{NB}\delta f_z \\
&\quad + f_n\varepsilon\theta - f_e\varepsilon\phi + \delta g - 2\omega_d v_e\varepsilon\lambda \\
&\quad + \frac{\left(v_e{}^2 + v_n{}^2\right)}{R_e{}^2}\varepsilon h - \frac{2v_n}{R_e}\varepsilon v_n - \left(\frac{2v_e}{R_e} + 2\omega_n\right)\varepsilon v_e
\end{aligned}
\tag{1}
$$

In the above equation, the term δg can be neglected compared to other terms because the change in the magnitude of gravity relative to changes in velocity and position error is very small. The attitude error equations are as Eq. 2.

$$
\begin{aligned}
\varepsilon\dot{\phi} &= \omega_d\varepsilon\lambda - \frac{v_e}{R_e{}^2}\varepsilon h + \frac{1}{R_e}\varepsilon v_e + \left(\omega_d - \frac{v_e\tan\lambda}{R_e}\right)\varepsilon\theta \\
&\quad + \frac{v_n}{R_e}\varepsilon\psi - T_{11}^{NB}\delta\omega_x - T_{12}^{NB}\delta\omega_y - T_{13}^{NB}\delta\omega_z \\
\varepsilon\dot{\theta} &= \frac{v_n}{R_e{}^2}\varepsilon h - \frac{1}{R_e}\varepsilon v_n + \left(\frac{v_e\tan\lambda}{R_e} - \omega_d\right)\varepsilon\phi \\
&\quad + \left(\omega_n + \frac{v_e}{R_e}\right)\varepsilon\psi - T_{21}^{NB}\delta\omega_x - T_{22}^{NB}\delta\omega_y - T_{23}^{NB}\delta\omega_z \\
\varepsilon\dot{\psi} &= -\left(\omega_n + \frac{v_e\sec^2\lambda}{R_e}\right)\varepsilon\lambda + \frac{v_e\tan\lambda}{R_e{}^2}\varepsilon h \\
&\quad - \frac{\tan\lambda}{R_e}\varepsilon v_e - \frac{v_n}{R_e}\varepsilon\phi - \left(\omega_n + \frac{v_e}{R_e}\right)\varepsilon\theta \\
&\quad - T_{31}^{NB}\delta\omega_x - T_{32}^{NB}\delta\omega_y - T_{33}^{NB}\delta\omega_z
\end{aligned}
\tag{2}
$$

The next crucial step is to isolate the sensor bias terms (δf and δω) from the error dynamics equations. By algebraically rearranging the terms in Equations (1) and (2), we can express these biases directly in terms of the system's state errors (like $\varepsilon\dot{v}$ and $\varepsilon\dot{\psi}$) and the transformation matrix (T^{NB}). This manipulation is the key to creating a solvable system, leading to the following formulations:

$$
\begin{aligned}
&T_{11}^{NB}\delta f_x + T_{12}^{NB}\delta f_y + T_{13}^{NB}\delta f_z = \varepsilon\dot{v}_n \\
&\quad - \begin{pmatrix} f_e\varepsilon\psi - f_d\varepsilon\theta - \left(\frac{v_e{}^2\sec^2\lambda}{R_e} + 2\omega_n v_e\right)\varepsilon\lambda \\ + \frac{\left(v_e{}^2\tan\lambda - v_n v_d\right)}{R_e{}^2}\varepsilon h + \frac{v_d}{R_e}\varepsilon v_n \\ + \left(2\omega_d - \frac{2v_e\tan\lambda}{R_e}\right)\varepsilon v_e + \frac{v_n}{R_e}\varepsilon v_d \end{pmatrix} \\
&T_{21}^{NB}\delta f_x + T_{22}^{NB}\delta f_y + T_{23}^{NB}\delta f_z = \varepsilon\dot{v}_e \\
&\quad - \begin{pmatrix} -f_n\varepsilon\psi + f_d\varepsilon\phi + \left(\frac{v_e v_n\sec^2\lambda}{R_e} + 2\omega_n v_n + 2\omega_d v_d\right)\varepsilon\lambda \\ - \frac{\left(v_e v_n\tan\lambda + v_e v_d\right)}{R_e{}^2}\varepsilon h + \left(\frac{v_e\tan\lambda}{R_e} - 2\omega_d\right)\varepsilon v_n \\ + \frac{\left(v_n\tan\lambda + v_d\right)}{R_e}\varepsilon v_e + \left(\frac{v_e}{R_e} + 2\omega_n\right)\varepsilon v_d \end{pmatrix} \\
&T_{31}^{NB}\delta f_x + T_{32}^{NB}\delta f_y + T_{33}^{NB}\delta f_z = \varepsilon\dot{v}_d \\
&\quad - \begin{pmatrix} + f_n\varepsilon\theta - f_e\varepsilon\phi - 2\omega_d v_e\varepsilon\lambda + \frac{\left(v_e{}^2 + v_n{}^2\right)}{R_e{}^2}\varepsilon h \\ - \frac{2v_n}{R_e}\varepsilon v_n - \left(\frac{2v_e}{R_e} + 2\omega_n\right)\varepsilon v_e \end{pmatrix}
\end{aligned}
\tag{3}
$$

$$
T_{11}^{NB}\delta\omega_x + T_{12}^{NB}\delta\omega_y + T_{13}^{NB}\delta\omega_z \tag{4}
$$

$$-\frac{v_e}{{R_e}^2}\varepsilon h+\frac{1}{R_e}\varepsilon v_e+\left(\omega_d-\frac{v_e\tan\lambda}{R_e}\right)\varepsilon\theta+\frac{v_n}{R_e}\varepsilon\psi$$

$$T_{21}^{\mathrm{NB}}\delta\omega_x+T_{22}^{\mathrm{NB}}\delta\omega_y+T_{23}^{\mathrm{NB}}\delta\omega_z=-\varepsilon\dot{\theta}+\frac{v_n}{{R_e}^2}\varepsilon h-\frac{1}{R_e}\varepsilon v_n+\left(\frac{v_e\tan\lambda}{R_e}-\omega_d\right)\varepsilon\phi+\left(\omega_n+\frac{v_e}{R_e}\right)\varepsilon\psi$$

$$T_{31}^{\mathrm{NB}}\delta\omega_x+T_{32}^{\mathrm{NB}}\delta\omega_y+T_{33}^{\mathrm{NB}}\delta\omega_z=-\varepsilon\dot{\psi}+\frac{v_e\tan\lambda}{{R_e}^2}\varepsilon h-\left(\omega_n+\frac{v_e\sec^2\lambda}{R_e}\right)\varepsilon\lambda-\frac{\tan\lambda}{R_e}\varepsilon v_e-\frac{v_n}{R_e}\varepsilon\phi-\left(\omega_n+\frac{v_e}{R_e}\right)\varepsilon\theta$$

Based on the above relationships, if the terms on the right side of the equations and the body-to-navigation frame transformation matrix (T^{NB}) are known, the accelerometer biases (δf) and gyroscope biases (δω) can be obtained by solving a system of three equations with three unknowns. Therefore, in a system with a reference and a dependent navigation system, assuming the axes of the dependent system are aligned with the axes of the reference system, the biases of the accelerometers and gyroscopes of the dependent system can be calculated algebraically using the state errors and equations (3) and (4).

2.2 Justification for the Algebraic Methodology

The selection of an algebraic approach for online bias estimation was driven by the need to overcome specific limitations inherent in conventional methods, primarily Kalman filters and data-driven techniques. The theoretical rationale for this choice is based on the following analysis:

- Computational Efficiency and Real-Time Performance: Filter-based methods like the EKF and UKF rely on recursive prediction and update cycles, which incur a significant and constant computational load. In safety-critical real-time systems, this can be a bottleneck. The proposed algebraic method, in contrast, is designed to provide a bias estimate through a direct, deterministic computation—essentially solving a system of linear equations. This non-iterative nature eliminates the convergence delay of filters and offers a fundamentally faster and more predictable execution time.
- Elimination of Tuning and Initialization Parameters: A well-known challenge in applying Kalman filters is the need for careful tuning of process and measurement noise covariance matrices (Q and R). Poor tuning can lead to divergence or inaccurate estimates. The algebraic method is entirely deterministic and requires no such tuning parameters. Its performance is solely dependent on the accuracy of the instantaneous state error measurements, making it more robust and easier to implement.
- Avoidance of Data-Dependency and Training: While machine learning methods offer powerful pattern recognition, they require extensive training datasets that cover all potential operational scenarios. This can be impractical for systems operating in unpredictable environments. The algebraic method is model-based and requires no prior training; it performs estimation based on the instantaneous physical relationship between state errors and sensor biases, as derived from the fundamental navigation equations.

Therefore, the algebraic method was chosen not merely as an alternative, but as a targeted solution for applications where computational simplicity, deterministic reliability, and elimination of tuning procedures are paramount. Its theoretical foundation provides a direct and efficient pathway from observable state errors to the underlying sensor biases, fulfilling a clear niche that other methods do not address optimally.

2.3 Objectives and Performance Analysis

The primary objective of this study is to develop and validate a novel algebraic method for the real-time estimation of inertial sensor biases that fulfills the following specific goals:

1) To derive a direct, closed-form solution for accelerometer and gyroscope biases from the

INS error dynamics, thereby eliminating the dependency on iterative estimation techniques like the Kalman filter.

2) To achieve computational efficiency by avoiding the complex matrix operations and parameter tuning required in probabilistic methods, making the algorithm suitable for real-time applications.
3) To ensure high estimation accuracy across different dynamic profiles, demonstrating the method's robustness not only in a simple stationary condition but also under more challenging cruise flight scenarios.
4) The proposed model demonstrates superiority over the conventional approach in the following criteria:
   - Estimation Accuracy: The closeness of the estimated biases to the true predefined values applied in the simulation.
   - Convergence Speed: The time required for the estimated biases to reach and remain within a small tolerance band of their final values.
   - Computational Efficiency: The algorithm's ability to provide estimates in a single computational step without iterative loops.

2.4 State Error Calculation

To provide complete transparency and facilitate reproducibility, this section details the step-by-step computational procedure of the proposed algebraic bias estimation method. The algorithm operates in real-time, utilizing instantaneous outputs from both reference and dependent navigation systems, as illustrated in Fig. 2.

According to [7] and assuming that the operators δ and ε are equivalent for the position and velocity state variables, we have:

$$\begin{aligned} &\varepsilon\lambda \triangleq \Delta\lambda = \lambda_m - \lambda_s \\ &\varepsilon l \triangleq \Delta l = l_m - l_s \\ &\varepsilon h \triangleq \Delta h = h_m - h_s \end{aligned} \tag{5}$$

$$\begin{aligned} &\varepsilon v_{\mathrm{n}} \triangleq \Delta v_{\mathrm{n}} = v_{\mathrm{n}_m} - v_{\mathrm{n}_s} \\ &\varepsilon v_e \triangleq \Delta v_e = v_{e_m} - v_{e_s} \\ &\varepsilon v_d \triangleq \Delta v_d = v_{d_m} - v_{d_s} \end{aligned} \tag{6}$$

In the above equations, subscript *m* refers to the reference navigation system and subscript *s* refers to the dependent system. Furthermore, based on [16], the body attitude error is obtained as:

$$\begin{aligned} \varepsilon\phi &= \Delta\theta\sin(\psi_s) - \Delta\phi\cos(\psi_s)\cos(\theta_s) \\ &\quad - \Delta\psi\,\Delta\theta\cos(\psi_s) - \Delta\phi\,\Delta\psi\cos(\theta_s)\sin(\psi_s) \\ \varepsilon\theta &= -\Delta\theta\cos(\psi_s) - \Delta\phi\cos(\theta_s)\sin(\psi_s) \\ &\quad - \Delta\phi\,\Delta\theta\sin(\psi_s)\sin(\theta_s) \\ \varepsilon\psi &= \Delta\phi\sin(\theta_s) - \Delta\psi - \Delta\phi\,\Delta\theta\cos^2(\psi_s)\cos(\theta_s) \\ &\quad - \Delta\phi\,\Delta\theta\,\Delta\psi\cos(\psi_s)\cos(\theta_s)\sin(\psi_s) \end{aligned} \tag{7}$$

The quantities Δϕ, Δθ and Δψ are defined as:

$$\begin{aligned} &\Delta\phi = \phi_m - \phi_s \\ &\Delta\theta = \theta_m - \theta_s \\ &\Delta\psi = \psi_m - \psi_s \end{aligned} \tag{8}$$

2.5 Algorithmic Implementation and Computational Procedure

The practical implementation of the proposed algebraic method follows a systematic computational procedure that transforms the theoretical derivations into an executable algorithm. The process begins with the simultaneous acquisition of navigation state vectors from both reference and dependent systems at each sampling interval. These vectors encompass position, velocity, and attitude parameters from both systems, providing the fundamental inputs for subsequent calculations.

Following data acquisition, the algorithm proceeds to calculate the complete set of navigation state errors. The position and velocity errors are computed directly through simple differences between corresponding states of the reference and dependent systems. For attitude errors, the algorithm employs the nonlinear relationships defined in Equation (7), which properly account for the trigonometric nature of orientation discrepancies. This comprehensive error calculation establishes the foundation for bias estimation.

The transformation matrix is then constructed using the reference system's Euler angles through standard direction cosine matrix formulation. This matrix serves as the crucial link between the body frame and navigation frame, enabling proper coordination transformation throughout the estimation process.

With the transformation matrix established and state errors calculated, the algorithm forms the right-hand side vectors δf and δω using the algebraic expressions derived in Equations (3) and (4). These vectors incorporate the calculated state

errors along with reference system parameters and necessary Earth rotation corrections, effectively encoding all required information for bias determination.

The core computational step involves solving for sensor biases through direct matrix operations. Leveraging the orthogonal property of the transformation matrix, the accelerometer and gyroscope biases are obtained simultaneously via straightforward matrix multiplication. This direct approach eliminates the need for iterative solutions and ensures deterministic execution time, making it particularly suitable for real-time applications.

For applications where measurement noise is significant, an optional filtering step can be implemented through temporal averaging over a short window. This simple yet effective approach enhances estimation stability without introducing the computational overhead associated with more complex filtering techniques.

The entire procedure is characterized by its computational efficiency, requiring only basic matrix operations and algebraic computations. This stands in stark contrast to filter-based approaches that involve iterative convergence, covariance propagation, and complex matrix inversions. The deterministic nature of the algorithm and its minimal computational requirements make it ideally suited for implementation in resource-constrained embedded systems while maintaining the accuracy required for demanding navigation applications.

3. MODELING

To model the problem, two free Inertial Navigation System (INS) models are considered. First model represents the reference navigation system, whose inputs are the true linear accelerations and angular velocities of the vehicle. Second model represents the dependent navigation system, whose inputs are the linear accelerations and angular velocities measured by the inertial measurement unit (IMU). The simulation framework, designed in Simulink, serves as the source for all data used in this study.

The outputs of the IMU include both biases and white noise.

Figure 1 illustrates the modeling of the reference and dependent system inputs.

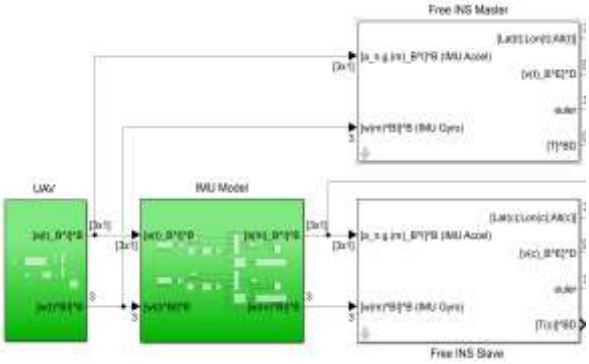


Fig. 1. Modeling of reference and dependent system inputs.

The outputs of both the reference and dependent system models include position, velocity, and Euler angles. These outputs are then fed into the bias estimation algorithm. Based on the equations described in the previous section, the algorithm estimates the accelerometer and gyroscope biases.

Figure 2 shows the outputs of the reference and dependent systems, along with the inputs to the bias estimation algorithm.

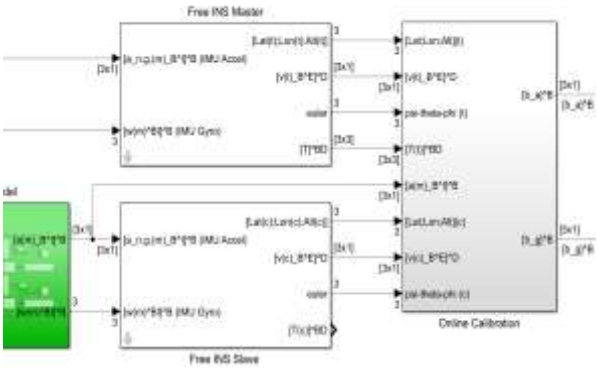


Fig. 2. Outputs of the reference and dependent systems and inputs to the bias estimation algorithm.

4. SIMULATION SCENARIOS AND RESULTS

The data for the analysis was generated entirely from the simulation model described in Section 3. The characteristics of the inertial sensor data are defined by the deterministic biases and stochastic noise parameters specified in Tables 1 and 2, which were superimposed on the true vehicle trajectory.

To validate the proposed method, two simulation scenarios are considered. In both scenarios, the accelerometer and gyroscope biases are set as follows.

Table 1. Applied accelerometer and gyroscope bias specifications.

	Sensor Type	Axis	Value	Unit
1	accelerometer	x	1e-3	$\frac{m}{s^2}$
2	accelerometer	y	2e-3	$\frac{m}{s^2}$
3	accelerometer	z	3e-3	$\frac{rad}{s}$
4	gyroscope	x	1e-8	$\frac{rad}{s}$
5	gyroscope	y	2e-8	$\frac{rad}{s}$
6	gyroscope	z	3e-8	$\frac{rad}{s}$

Additionally, zero-mean Gaussian noise is added to the IMU accelerometer and gyroscope outputs. The noise characteristics are given in Table 2.

Table 2. Applied accelerometer and gyroscope bias specifications.

	Sensor Type	Mean	Noise Power	Unit
1	accelerometer	0	1e-5	$\frac{m}{s^2}$
2	gyroscope	0	1e-10	$\frac{rad}{s}$

4.1 Scenario 1: Stationary Condition

The conditions for this scenario are listed in Table 3.

Table 3. Scenario 1 conditions.

	Parameter	Value	Unit
1	Initial latitude	35.722	deg
2	Initial longitude	51.335	deg
3	Initial altitude	0	m
4	Initial north velocity	0	$\frac{m}{s}$
5	Initial east velocity	0	$\frac{m}{s}$
6	Initial down velocity	0	$\frac{m}{s}$
7	Initial roll angle	0	deg
8	Initial pitch angle	30	deg
9	Initial yaw angle	45	deg
10	Specific force (x-body)	0	$\frac{m}{s^2}$
11	Specific force (y-body)	0	$\frac{m}{s^2}$
12	Specific force (z-body)	0	$\frac{m}{s^2}$
13	Angular rate (x-body)	5.738e-5	$\frac{rad}{s}$
14	Angular rate (y-body)	-4.17e-5	$\frac{rad}{s}$
15	Angular rate (z-body)	-1.59e5	$\frac{rad}{s}$

The results of the bias results in this scenario are shown in Figs. 3, 4 and Table 4.

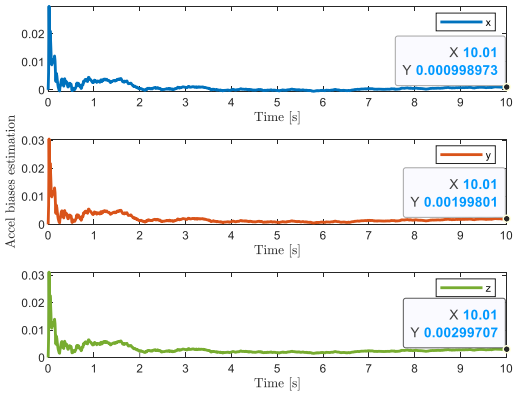


Fig. 3. Estimated accelerometer biases in scenario 1.

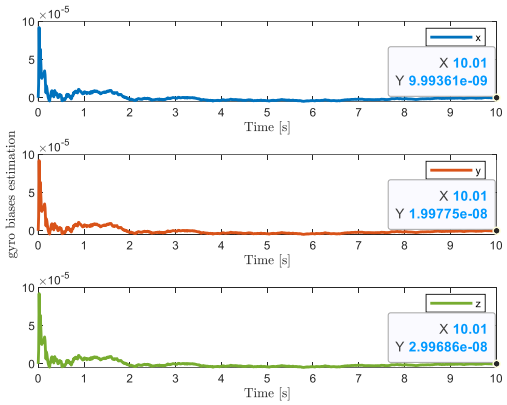


Fig. 4. Estimated gyroscope biases in scenario 1.

Table 4. Bias estimation results for accelerometers and gyroscopes in scenario 1.

	Sensor Type	Axis	Estimated Bias	Error (%)
1	accelerometer	x	9.98973e-4	0.1027
2	accelerometer	y	0.00199873	0.0997
3	accelerometer	z	0.00299707	0.0978
4	gyroscope	x	9.99361e-9	1.1206
5	gyroscope	y	1.99775e-8	0.1734
6	gyroscope	z	2.99686e-8	0.7412

The results presented in Table 4 and Fig. 3 and 4 demonstrate the high accuracy of the proposed algorithm. The very low estimation errors—all below 0.11% for accelerometers and approximately 1.12% for gyroscopes—validate the method's effectiveness in a stationary scenario. The

slightly higher relative error for the gyroscope biases is expected, as their smaller magnitude makes them more susceptible to the influence of the simulated Gaussian noise. Furthermore, the convergence plots in Figs. 3, 4 show that the estimates stabilize rapidly after the algorithm is initiated. This fast convergence is a key advantage, indicating that the method does not require a lengthy initialization period and is suitable for applications requiring quick calibration.

4.2 Scenario 2: Cruise Flight

The conditions for this scenario are listed in Table 5.

Table 5. Scenario 2 conditions.

	Parameter	Value	Unit
1	Initial latitude	35.722	deg
2	Initial longitude	51.334	deg
3	Initial altitude	3000	m
4	Initial north velocity	150	$\frac{m}{s}$
5	Initial east velocity	150	$\frac{m}{s}$
6	Initial down velocity	0	$\frac{m}{s}$
7	Initial roll angle	0	deg
8	Initial pitch angle	30	deg
9	Initial yaw angle	45	deg
10	Specific force (x-body)	0	$\frac{m}{s^2}$
11	Specific force (y-body)	0	$\frac{m}{s^2}$
12	Specific force (z-body)	0	$\frac{m}{s^2}$
13	Angular rate (x-body)	5.738e-5	$\frac{rad}{s}$
14	Angular rate (y-body)	-4.17e-5	$\frac{rad}{s}$
15	Angular rate (z-body)	-1.59e5	$\frac{rad}{s}$

The results of the bias results in this scenario are shown in Figs. 5, 6 and Table 6.

The results for the cruise flight scenario, detailed in Table 6 and Figs. 5, 6, are critically important as they validate the method's performance under dynamic conditions. The consistency in estimation accuracy, with errors nearly identical to those in the stationary case, strongly indicates that the algorithm is robust and not significantly affected by vehicle dynamics. This robustness is a primary finding, as it confirms that the algebraic formulation successfully isolates the sensor biases from the navigation state errors even in a more challenging and realistic operational environment. The maintained convergence speed further underscores its suitability for real-time applications during actual mission profiles.

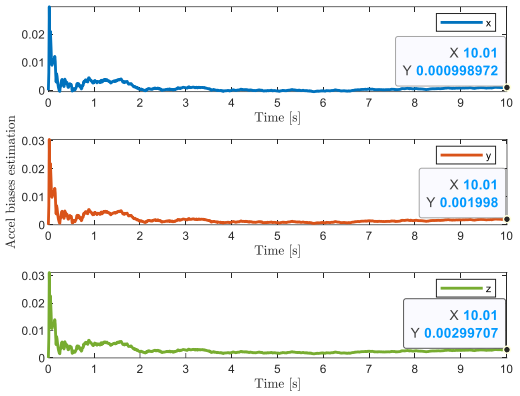


Fig. 5. Estimated accelerometer biases in scenario 2.

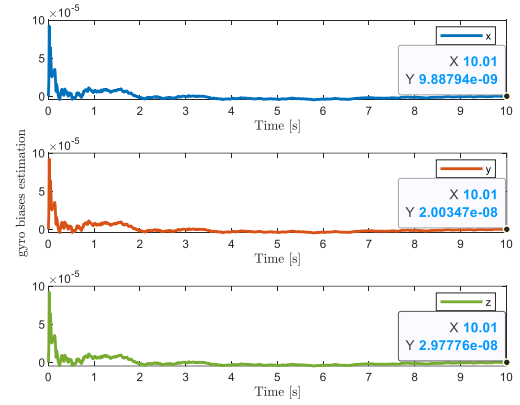


Fig. 6. Estimated gyroscope biases in scenario 2.

Table 6. Bias estimation results for accelerometers and gyroscopes in scenario 2.

	Sensor Type	Axis	Estimated Bias	Error (%)
1	Accelerometer	x	9.9897e-4	0.1028
2	Accelerometer	y	0.001998	0.0998
3	Accelerometer	z	0.002997	0.0977
4	Gyroscope	x	9.988794e-9	1.1206
5	Gyroscope	y	2.00347e-8	0.1734
6	Gyroscope	z	2.97776e-8	0.7412

4.3 Discussion of Results

The simulation results from both scenarios provide compelling evidence for the effectiveness of the proposed algebraic method through several key aspects:

1) Consistent Accuracy Across Dynamic Conditions: The method maintains nearly identical estimation accuracy (errors < 0.11% for accelerometers, < 1.2% for gyroscopes) in both stationary and cruise flight conditions. This consistency demonstrates the method's robustness to different dynamic profiles, a critical requirement for practical navigation applications.
2) Instantaneous Convergence Property: As clearly visible in Figs. 3-6, the estimated biases converge to their true values within the first computation step. This immediate convergence contrasts sharply with iterative methods like Kalman filters that require multiple time steps to reach steady-state estimates. The algebraic approach achieves this by solving for biases directly rather than through recursive estimation.
3) Computational Efficiency Demonstrated: The method's computational advantage is inherent in its formulation. By transforming the bias estimation problem into a deterministic system of linear equations solvable through direct matrix operations, it avoids the computationally expensive prediction-update cycles, covariance propagation, and parameter tuning required by filter-based approaches. The consistent real-time performance across both scenarios validates this efficiency.
4) Noise Immunity Through Structure: The algorithm's structure naturally provides some noise attenuation, as evidenced by the stable estimates despite the presence of Gaussian measurement noise. This is achieved through the mathematical formulation that isolates bias terms from stochastic disturbances in the derived equations.

These characteristics—consistent accuracy, immediate convergence, computational simplicity, and inherent noise handling—collectively demonstrate the method's effectiveness and represent significant advantages over conventional approaches for real-time calibration applications.

5. CONCLUSION

This paper has presented a novel algebraic method for the real-time estimation of accelerometer and gyroscope biases in reference-dependent inertial navigation systems. The core innovation lies in the reformulation of the INS error dynamics into a deterministic system of linear equations, enabling the direct calculation of sensor biases without recursive filtering. Simulation results under both stationary and cruise flight conditions demonstrated the method's high accuracy and rapid convergence, with estimation errors of less than 0.11% for accelerometers and approximately 1.2% for gyroscopes.

The primary advantage of this approach is its computational efficiency. By avoiding the iterative loops and complex matrix operations of filter-based methods, it achieves a significant reduction in processing load, making it suitable for systems with limited computational resources.

Notwithstanding these advantages, the current work has several limitations. The proposed method is inherently dependent on the availability of an accurate reference navigation system, as errors in the reference will directly propagate into the bias estimates. Furthermore, the current model assumes perfect alignment between the reference and dependent systems' body frames, and its performance was validated in a simulation environment with known and constant biases. The algebraic solution, while efficient, is also more sensitive to high-frequency noise in the state error measurements compared to recursive estimators that have inherent filtering capabilities.

These limitations point toward clear directions for future work. The potential for future improvements in scalability and processing efficiency could be explored through the use of optimized linear algebra libraries or hardware acceleration. Future research will also focus on extending the method to account for sensor misalignment, validating its performance with real-world experimental data under complex error sources, and investigating its integration into a hybrid framework for enhanced robustness and noise immunity.

In summary, the proposed method provides a fast, accurate, and computationally efficient alternative for real-time bias estimation, with a clear pathway for further development and application in demanding aerospace, automotive, and robotic systems.

CONFLICT OF INTERESTS

No conflict of interest has been expressed by the authors.

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