

Original Research Paper

A Correction on the Lambert Targeting Problem in the Perturbed Space Environment

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ABSTRACT

This article introduces a novel correction algorithm designed to enhance the Lambert targeting problem (LTP) by addressing space perturbations. The proposed LTP correction (LTPC) algorithm integrates the traditional LTP with the shooting method and particle swarm optimization (PSO) to improve targeting accuracy. Following an initial solution using the LTP, the LTPC employs five sequential and iterative refinement stages to converge on a more precise result. The LTPC modifies the classical LTP to incorporate space perturbations, with Earth's oblateness being the primary perturbation considered. However, the algorithm is versatile and can be adapted to account for various types of space perturbations, broadening its applicability. The results demonstrate that the delta true anomaly, the difference between the desired arrival true anomaly and the actual arrival true anomaly, achieved by the LTPC is significantly smaller than that of the traditional LTP. This improvement highlights the LTPC's superior targeting accuracy. Notably, the LTPC achieves this enhanced precision without requiring a substantial increase in flight time or total velocity change compared to the classical LTP. This balance of accuracy and efficiency underscores the LTPC's effectiveness as a robust solution for space missions influenced by perturbations, paving the way for more reliable mission planning and execution.

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1. INTRODUCTION

Lambert Targeting Problem (LTP) problem is the classical two-point boundary value problem (TPBVP) in celestial mechanics, that was first posed and solved by Johann Heinrich Lambert in 1761 [1]. It is known to have a unique solution for the fractional orbit transfer between prescribed positions in a prescribed “time of flight.” Solving the problem requires determining the orbital arc (typically, solving for the initial velocity) connecting a prescribed initial position and final position, which correspond to the specified flight time. Many investigations established the LTP. A uniform solution to the Lambert problem is investigated by Kriz [2]. Lambert problem solution in the hill model of motion by Sukhanov and Prado [3]. They explain the LTP in the Lambert problem in the restricted three-body problem. Leeghim and Jaroux studied the Energy-optimal solution to the Lambert problem [4]. New solutions for the Lambert problem using regularization and Picard iteration are done by Woollands and Younes [5]. Thompson and Rostowfske express practical constraints for the applied Lambert problem [6]. Constrained multiple-revolution Lambert’s problem is investigated by Zhang and Moratari. A fixed-time, multiple-revolution Lambert’s problem is solved under given constraints in their study. A solution based on a dynamical approach for the multiple-revolution Lambert problem is investigated by Arlulkar and Naik [7]. Li and Han explained multiple-revolution solutions of the transverse-eccentricity-based Lambert problem [8]. The Uncertain Lambert problem with a probabilistic approach is explained by Adurthi and Majji [9]. All the mentioned research established the LTB ignoring the effects of space perturbations. But it should be noted that the effect of perturbations, however small, can be the cause of the success or failure of a space mission. Therefore, A novel correction algorithm is presented in this article to modify the Lambert targeting problem (LTP). To take into consideration space perturbations, the proposed LTPC (LTP Correction) algorithm combines the LTP with the shooting method [10], [11] and Particle Swarm Optimization (PSO) [12], [13]. Once the LTP is applied, the five consecutive iterative stages are established to modify it. The novel developed algorithm is called LTPC, which modifies LTP to regard space perturbations. The Earth’s oblateness

is considered a perturbation of interest [14]–[16]. This is while the proposed method can regard any space perturbations. The results indicate that the proposed LTPC has a much higher targeting accuracy, which is due to the ability to consider space perturbations

2. LAMBERT TARGETING PROBLEM (LTP)

Briefly, the Lambert Targeting problem (LTP) determines the transfer orbit between the initial orbit at the departure point $p_1(\vec{r}_d, \vec{v}_d)$ and final orbit at the arrival point $p_2(\vec{r}_a, \vec{v}_a)$ by having transfer time $t_{lambert}$ [4], [17], [18]. In fact, LTP finds the desired velocity vector $(\vec{V}_{LTP_d}, \vec{v}_{LTP_r})$ of transfer orbit at \vec{r}_d, \vec{r}_a . The orbital elements of transfer orbit are also obtained from the state vector at $(\vec{r}_a, \vec{V}_{LTP_a})$ or $(\vec{r}_a, \vec{V}_{LTP_d})$ [19]. The required total velocity changes for the deployment of a satellite equals:

$$\Delta \vec{V}_{Sat1} = |\vec{V}_{LTP_a} - \vec{v}_a| + |\vec{V}_{LTP_d} - \vec{v}_d| \quad (1)$$

One of the most critical requirements of designing a mission is trying to make it operational. LTP cannot be considered as a practical scheme in the mission design due to the following disadvantages:

- LTP cannot consider space perturbations due to the use of Kepler’s equations of motion.
- Due to the accumulative trait of the perturbations, LTP is not suitable for long-term missions.

Therefore, since accurate targeting is vital in the orbital transferring and rendezvous maneuvers, the LTP is not appropriate for practical space missions.

Suppose that the Earth’s oblateness is chosen as the space perturbations of interest in this research. The perturbed equations of motion due to the oblate perturbations are as follows [20]:

$$\begin{aligned} \ddot{x} &= -\frac{\mu}{r^3}x + \frac{3J_2\mu R^2}{r^4} \left[\frac{x}{r} \left(\frac{5z^2}{r^2} - 1 \right) \right] \\ \ddot{y} &= -\frac{\mu}{r^3}y + \frac{3J_2\mu R^2}{r^4} \left[\frac{y}{r} \left(\frac{5z^2}{r^2} - 1 \right) \right] \\ \ddot{z} &= -\frac{\mu}{r^3}z + \frac{3J_2\mu R^2}{r^4} \left[\frac{z}{r} \left(\frac{5z^2}{r^2} - 3 \right) \right] \end{aligned} \quad (2)$$

In Eq. (2), the Earth's radius and the distance from the Earth's center to the center of the satellites are denoted by R and r , respectively. J_2 is the 2nd zonal harmonics of the Earth and zonal harmonics less than j_2 have been ignored [21]. The position and velocity vector of the satellite at the Earth-centered, Earth-coordinate frame (ECEF) is denoted by $\mathbf{X} = [x, y, z]$ and $\mathbf{V} = [\dot{x}, \dot{y}, \dot{z}]$, respectively. As an example, assume that the classic orbital elements of the initial and the final orbit are given in Table 1. The orbit's semi-major axis, eccentricity, inclination, right ascension of the ascending node, and the argument of perigee are denoted by a, e, i, Ω, ω .

Table 1. The classic orbital elements of the initial and the final orbit.

Orbit	$a(km)$	e	$i(deg)$	$\Omega(deg)$	$\omega(deg)$
Initial	8000	0.25	25	25	8
Final	21500	0.09	25	25	8

Assume that the true anomaly of the arrival points θ_a are given in the mission profile as: $\theta_a = 60^\circ$. The flight time of the mission t and departure θ_d are selected as design parameters and selected by the optimization algorithm. This paper suggested the Particle Swarm Optimization (PSO) algorithm as the optimization algorithm and the Eq. (2) is selected as an objective function that could minimize the total velocity change of the maneuver and consequently the fuel consumption. The process of the PSO is fully described in section 3. The LTP is established in Fig. (1) once due to unperturbed and once to perturbed orbital equations of motion. Note that by setting the perturbation terms to zero in Eq. (2), the unperturbed equations will be obtained.

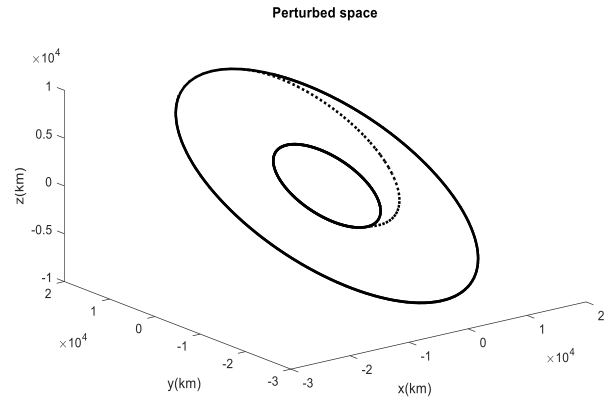
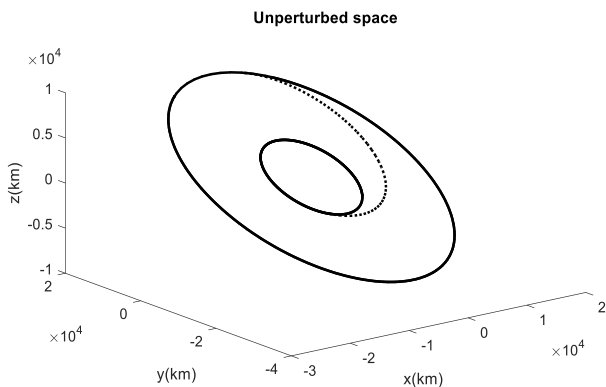


Fig. 1. The LTP maneuver once due to unperturbed and once to perturbed orbital equations of motion.

As mentioned the desired true anomaly was $\theta_a = 60^\circ$. However, according to Fig. 1, the effect of oblate perturbations has caused the satellite to arrive at the final orbits in $\theta_a = 69.8^\circ$. Therefore, according to Fig. 1, the inability of the LTP to consider perturbations has caused a significant decrease in targeting accuracy. Therefore, this paper proposed an approach to modify the LTP to consider space perturbations and accurate targeting. In section 3, the PSO optimization algorithm will be described, and section 4 will deal with the development of LTP Correction (LTPC).

3. PSO OPTIMIZATION

In computational science, particle swarm optimization (PSO) is a computational method that optimizes a problem by iteratively trying to improve a candidate solution with regard to a given measure of quality. First, a finite number of seeds is scattered randomly in the D-dimensional search space, representing the optimization problem's initial solutions. Then according to the seeds' fitness value, the member of the colonies is ranked and sorted. The i -th plant, which represents the i -th initial position, is shown by $X^{i,best}$. An $X^{i,best}$ which has the most satisfying fitness value is the best position of the colony, and is shown by $X^{g,best}$. Updating the speed and position of each particle is obtained from the following equations [22-26].

$$v^{i,s}[t + 1] = \gamma V^i[t] + c_1 r_{1,s}(X^{i,best}[t] - X^i[t]) + c_2 r_{2,s}(X^{g,best}[t] - X^i[t]) \quad (3)$$

$$X^{i,s}[t + 1] = X^i[t] + V^{i,s}[t + 1] \quad (4)$$

The inertia weight γ may help to keep the motion of each particle in its main direction and balance the global and local search. Also, in Eqs. (3) and (4), the velocity and position of the s -th seed of the i -th member shows by $V^{i,s}, X^{i,s}$ respectively.

The cognitive coefficient c_1 and social coefficient c_2 control the rate of convergence towards a personal best or global best location. $r_{1,s}$ and $r_{2,s}$ are the random numbers in the range $[0, 1]$. The flight time of the mission t and departure θ_d are selected as design parameters and selected by the PSO optimization algorithm.

4. LAMBERT TARGETING PROBLEM CORRECTION (LTPC)

In this article, an innovative correction algorithm is proposed to modify LTP. The proposed LTPC (LTP Correction) algorithm is developed by combining the LTP with the shooting method. The shooting method solves a two-boundary value problem by reducing it to an initial value problem system [14]. The orbital transfer mission is performed once applying the LTP. Then, the following iterative process is established to modify LTP. The newly developed algorithm is called LTPC, which modifies LTP to consider space perturbations:

1. The LTP is applied and optimized with the PSO for the satellite deployment mission.
2. The departure position vector of the LTP maneuver \vec{r}_d is chosen as an initial condition.
3. The new impulse velocity vector \vec{V}_{P_s} at \vec{r}_d and the new flight time t_s are also selected as an initial condition by PSO, and the shooting algorithm started (starting LTP correction).
4. The transfer orbit is determined by applying the system of Eq. (2) with the selected initial conditions in the previous steps.
5. Calculate the difference between the new position and the velocity vector of the satellite $(\vec{r}_{a_N}, \vec{v}_{a_N})$ relative to the values determined by the LTP (\vec{r}_a, \vec{v}_a) at the arrival point.
6. The LTP is replaced by the transfer orbit determined by the shooting algorithm (LTPC) if condition (5) is satisfied:

$$\begin{aligned} |\vec{r}_{a_N} - \vec{r}_{a_{MLTP}}| &< 2 \times 10^{-3} (m) \\ |\vec{v}_{a_N} - \vec{v}_{a_{MLTP}}| &< 10^{-3} \left(\frac{km}{s}\right) \end{aligned} \quad (5)$$

7-If the delta values do not meet condition (5), then return to step 3 and shoot again. In the results section, the correctness of the proposed algorithm will be validated.

5. RESULT

The main purpose of this article is to develop an orbital transfer that has accurate targeting and could regard space perturbation. For this reason, first, the LTP is suggested for accurate targeting. Since the LTP cannot consider space perturbations, this paper proposed a correction on the LTP that modified the LTP to regard space perturbations with accurate targeting. In this section, two cases study is investigated.

The classic orbital elements of the initial and the final orbit of these cases study are given in table 2.

Table 2. the classic orbital elements of the initial and the final orbit of the case study.

Orbit	$a(km)$	e	$i(deg)$	$\Omega(deg)$	$\omega(deg)$
Initial	9000	0.1	20	20	10
Final	23000	0.08	20	20	10

In the first case study, it is assumed that the satellite must be deployed at $\theta_a = 150^\circ$. The LTP is compared with the proposed LTPC in Fig. 2.

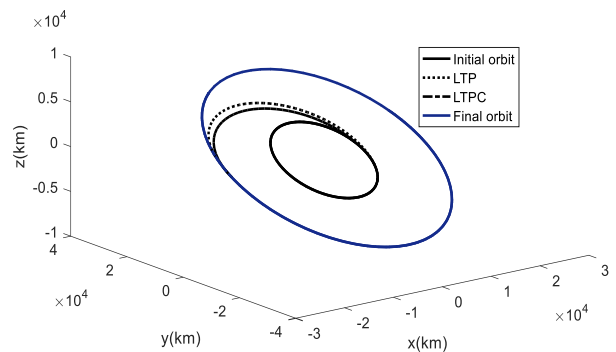


Fig. 2. The comparison between the LTP and proposed LTPC in the first case study.

The arrival true anomaly θ_a , the total velocity change ΔV_{tot} and the flight time t between the LTP and the proposed LTPC are compared in Table 3 for the first case study.

Table 3. the comparison of arrival true anomaly θ_a , the total velocity change ΔV_{tot} and the flight time t between the LTP and the proposed LTPC.

	$\theta_a(deg)$	$\Delta V_{tot} \left(\frac{km}{s}\right)$	$t(sec)$
LTP	161	2.18	10403
LTPC	153	2.21	10487

According to the table 2, the delta true anomaly between the desired true anomaly $\theta_a = 150^\circ$ and the arrival true anomaly LTP and proposed LTPC are 11° and 3° respectively. Therefore, the LTPC has accurate targeting with considering space perturbation. This holds despite the fact that the difference between the velocity change and flight time of the LTP and LTP are neglectable. In the first case study, it is assumed that the satellite must be deployed at $\theta_a = 225^\circ$. The LTP is compared with the proposed LTPC in the Fig. 3.

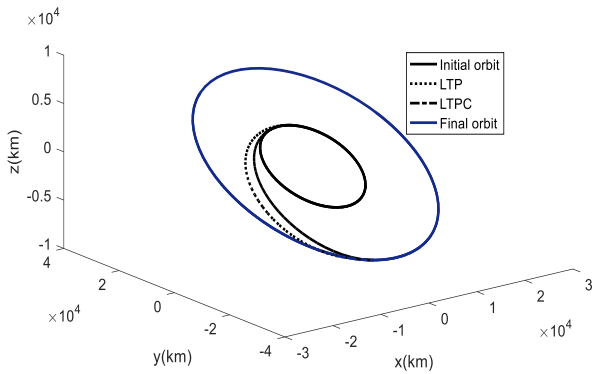


Fig. 3. The comparison between the LTP and proposed LTPC in the second case study.

The arrival true anomaly θ_a , the total velocity change ΔV_{tot} and the flight time t between the LTP and the proposed LTPC are compared in Table 4 for the first case study.

Table 4. the comparison of arrival true anomaly θ_a , the total velocity change ΔV_{tot} and the flight time t between the LTP and the proposed LTPC.

	$\theta_a(deg)$	$\Delta V_{tot} \left(\frac{km}{s}\right)$	$t(sec)$
L T P	237.4	2.32	11812
L T P C	227.5	2.35	11854

According to Table 4, the delta true anomaly between the desired true anomaly $\theta_a = 225$ and the arrival true anomaly LTP and proposed LTPC are 12.4° and 2.5° respectively. Therefore, like the first case study, the LTPC has accurate targeting considering the space perturbation and as illustrated in the first case study although the difference between the velocity change and flight time of the LTP and LTP are neglectable.

6. CONCLUSION

The primary objective of this paper was to develop an orbital transfer maneuver with high targeting accuracy, capable of accounting for all space perturbations, thus ensuring its practicality for real-world missions. The Lambert Targeting Problem (LTP) was initially proposed as an accurate orbit transfer maneuver. However, the LTP's inability to account for space perturbations significantly reduced its targeting accuracy, making it unsuitable for operational missions due to the following limitations:

Inability to account for space perturbations: The LTP relies on Kepler's equations of motion, which do not consider perturbative forces.

Unsuitability for long-term missions: The cumulative effect of perturbations over time renders the LTP ineffective for extended missions.

To address these shortcomings, this paper introduces an innovative correction algorithm, referred to as the Lambert Targeting Problem Correction (LTPC) algorithm. The LTPC combines the LTP with the shooting method to improve its performance. The orbital transfer maneuver begins with the application of the LTP, followed by five iterative stages designed to refine the solution. This process enables the LTPC to account for space perturbations effectively.

In this study, Earth's oblateness was considered the primary perturbation of interest. The results demonstrated that the LTPC achieved a significantly smaller delta true anomaly, the difference between the desired arrival true anomaly and the actual arrival true anomaly—compared to the classical LTP. Moreover, the LTPC maintained comparable flight times and total velocity change requirements to the LTP. These findings highlight the LTPC's superior targeting accuracy, attributed to its ability to incorporate space perturbations.

CONFLICT OF INTERESTS

No conflict of interest has been expressed by the authors.

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